The Law of General Average*

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Abstract

Part of a ship's cargo is jettisoned in order to save the vessel and the remaining cargo from imminent peril. How should the loss be shared among the cargo owners? The law of general average, an ancient principle of maritime law, prescribes that the owners share the loss proportionally according to the respective values of their cargo. We analyze whether the law of general average is a truthful and efficient mechanism. That is, we investigate whether it induces truthful reporting of cargo values and yields a Pareto efficient allocation in equilibrium. We show that the law of general average is neither truthful nor efficient if owners have expected utility preferences, but is both truthful and efficient if owners have maxmin utility preferences. We discuss why maxmin behavior may be reasonable in the general average context.

Keywords: general average, risk sharing, maritime law, maxmin, mutual insurance, truthful equilibrium, Pareto efficiency.

JEL classifications: C72, D82, G22, K39.

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1 Introduction

Part of a ship's cargo is voluntarily jettisoned in order to save the vessel and the remaining cargo from imminent peril. How should the loss be shared among the cargo owners?

The law of general average, an ancient principle of the general maritime law of nations, prescribes that the owners share the loss in proportion to the respective values of their cargo. Its roots can be traced back to a provision of Roman law, Digest XIV.2.1, which cites the Rhodian law of jettison: "The Rhodian law provides that if cargo has been jettisoned in order to lighten a ship, the sacrifice for the common good must be made good by common contribution" (Watson 1998, p. 419). Modern courts have interpreted this maxim to require pro rata contribution.[1] A contemporary statement of the law of general average, which is also known simply as general average, is set forth in *Zim Israel Navigation Co., Ltd. v. 3-D Imports, Inc.*, 29 F. Supp. 2d 186, 190 (S.D.N.Y. 1998) (citations omitted):

> "General Average is an ancient doctrine, referring to rules apportioning loss suffered by cargo owners whose goods are sacrificed in a maritime adventure. . . . [W]hen one partner in the adventure sacrifices its cargo or incurs expenses for the general safety of the ship and other cargo, the loss is assessed against all participants in proportion to their respective share in the adventure. Today, contribution in General Average is recognized by all major maritime nations."

Prior work by Landes and Posner (1978, pp. 106-108) showed that the law of general average has important efficiency properties. Their model, however, assumed cargo values are objective and public information, whereas they often are subjective and private information. Consequently, Landes and Posner analyzed only the incentives provided by the law of general average for the ship master's decisions regarding which and how much cargo to jettison. They showed that the general average principle gives the master the incentive to minimize the collective loss "by selecting the lowest-valued (per lb.) goods to jettison." But they did not consider the incentives it provides owners for truthful reporting of cargo values.

Epstein (1993, pp. 582-584) recognized that the "secret" to making the general average mechanism "work," in the sense of enabling the master to minimize the collective loss, is to get truthful reporting of cargo values.[2] (In fact, truthful reporting is sufficient, but not necessary, to enable the master to minimize the collective loss. What's necessary is *truthful ordering*, i.e., the owners' declared values must be in the same rank order as their true values.)

[1] See, e.g., *Cia. Atlantica Pacifica, S.A. v. Humble Oil & Refining Co.*, 274 F. Supp. 884, 891 (D. Md. 1967) ("The principle embodied in this maxim [is] that loss for the common benefit which is incurred by one who partakes in a maritime venture should be shared ratably by all who participate in the venture. . . ."). See also *Empire Stevedoring Co. v. Oceanic Adjusters, Ltd.*, 315 F. Supp. 921, 927 (S.D.N.Y. 1970).

[2] See also Gregory et al. (1977, pp. 35-36).

Epstein offered intuition for why the law of general average gives owners the right incentives for truthful reporting. However, he did not provide a formal game-theoretic treatment or welfare analysis of the general average mechanism.

We model the general average game and analyze whether the law of general average is a truthful and efficient mechanism. That is, we investigate whether the law of general average induces truthful reporting of cargo values and yields a Pareto efficient allocation in equilibrium. We show that truthful reporting is not a Bayesian Nash equilibrium of the general average game if owners have expected utility preferences, but that truthful reporting is the unique Nash equilibrium if owners have maxmin utility preferences. We further show that if owners have expected utility preferences, (i) the law of general average does not yield a Pareto efficient allocation in equilibrium because it does not induce truthful ordering (let alone truthful reporting) in equilibrium,[3] and (ii) even with truthful reporting, an allocation prescribed by the law of general average, which ipso facto entails proportional loss sharing, is Pareto efficient if and only if owners have identical (up to a positive scalar) CRRA utility functions.[4] If owners have maxmin utility preferences, by contrast, the law of general average not only induces truthful reporting, it also yields a Pareto efficient allocation.

In addition to contributing to the niche literature on the economics of general average, our paper makes contributions to two broader literatures in economics and law.

The first is the economics of mutual insurance. The paper most closely related to ours is Cabrales et al. (2003) which analyzes a mutual fire insurance mechanism in Andorra called *La Crema*. In the *La Crema* game, each homeowner reports the value of his house. In case of a fire, one or more houses burn down, where nature determines which and how many houses burn. Each owner of a burned house receives his reported value, which is paid by all homeowners (including himself) in proportion to their reported values. The general average game is similar. Each cargo owner declares the value of her cargo. In an emergency, cargo is jettisoned in ascending order of declared value, where nature determines how much cargo must be jettisoned. Each owner of jettisoned cargo receives her declared value, which is paid by all cargo owners (including herself) in proportion to their declared values. The key difference between the two games is that in the *La Crema* game whether an owner's house burns down is independent of the reported values, whereas in the general average game whether an owner's cargo is jettisoned depends on all the declared values including her own.

The second broader literature to which we contribute is the economics of ancient law. A prime example is Aumann and Maschler (1985) which presents a game-theoretic analysis

[3]Again, *truthful ordering* means that the owners' declared values have the same rank ordering as their true values. That is, the set of declared values is an order-preserving transformation of the set of true values.

[4]The acronym CRRA stands for constant relative risk aversion.

of a bankruptcy problem in the Babylonian Talmud and shows that the Talmudic solution coincides with the nucleolus of the corresponding coalitional game. The principle underlying the Talmudic solution is not proportional division. Still, the Talmudic bankruptcy problem and the general average problem are similar in that the core question is how to divide a residual among claimants whose claims sum to more than the total value of the residual. Miller (2010) collects two dozen contributions to this literature which cover a wide range of topics including ancient liability systems, family law, land law, and criminal law.

The remainder of the paper proceeds as follows. Section 2 is a brief primer on the law of general average. Section 3 describes the general average game. Sections 4 and 5 present our equilibrium results for the cases where cargo owners have expected utility preferences (the Bayesian game) and where owners have maxmin utility preferences (the maxmin game), respectively. Section 6 presents our efficiency results for both cases. Section 7 concludes the paper with a discussion in which we compare and contrast our results with those of Cabrales et al. (2003), suggest why maxmin behavior may be reasonable in the general average context, and point to directions for future research. The Appendix collects selected proofs.

2 The Law of General Average

In maritime law, the term "average" means damage or loss (Shoenbaum 2011, p. 253). It is an anglicization of the French nautical term *avarie*, which in turn derives from the Arabic word *'awār* through the Latin (and later Italian) *avaria* (Healy and Sharpe 1999, p. 760; Khalilieh 2006, p. 150). The term "general average" refers to a collective loss. It is the loss incurred when, for the benefit of all parties with a financial interest in the voyage, part of a ship or its cargo is voluntarily sacrificed to avoid a common imminent peril (Chamberlain 1940, p. 89). Under the law of general average, the parties share the collective loss in proportion to the values of their respective interests (Healy and Sharpe 1999, p. 761; Khalilieh 2006, p. 151).[5]

The principle embodied in the law of general average dates back to antiquity. The Babylonian Talmud, Bava Kamma 116b, articulates the principle in the context of land caravans: "If a caravan was traveling through the wilderness and a band of robbers threatened to plunder it, the apportionment [for buying them off] will have to be made according with the [value] of possessions [in the caravan]" (Friedell 1996, p. 656). A snippet of Roman law, Digest XIV.2.1, which references the Rhodian law of jettison, is the earliest statement of the principle in the maritime context: "The Rhodian law provides that if cargo has been jettisoned in order to lighten a ship, the sacrifice for the common good must be made good by common contribution" (Watson 1998, p. 419). Later statements appear in medieval

[5]See also, e.g., Rose (1997, p. 2) and Robertson et al. (2001, p. 426).

European maritime codes such as the Rolls of Oleron and the Laws of Wisby, and in early modern maritime codes such as the Guidon de la Mer and the Ordonnance de la Marine (Barclay 1891; Lowndes et al. 1912, pp. 4-16).[6]

By the turn of the nineteenth century, the law of general average had been incorporated into the English common law (Lowndes et al. 1912, pp. 18 & 21). Justice Lawrence of the Court of King's Bench gave the following definition in *Birkley v. Presgrave*, 1 East 220, 228 (1801): "All loss which arises in consequence of extraordinary sacrifices made or expenses incurred for the preservation of the ship and cargo comes within general average, and must be borne proportionably by all who are interested" (Lowndes et al. 1912, p. 21).[7]

In *McAndrews v. Thatcher*, 70 U.S. 347, 366 (1865), the United States Supreme Court defined the law of general average as follows:

> "[W]here two or more parties are in a common sea risk, and one of them makes a sacrifice or incurs extraordinary expenses for the general safety, the loss or expenses so incurred shall be assessed upon all in proportion to the share of each in the adventure; or, in other words, the owners of the other shares are bound to make contribution in the proportion of the value of their several interests."[8]

Three requirements must be met for a loss to qualify for general average contribution: "First, that the ship and cargo should be placed in a common imminent peril; secondly, that there should be a voluntary sacrifice of property to avert that peril; and, thirdly, that by that sacrifice the safety of the other property should be presently and successfully attained."[9] The archetypal general average case involves the jettison of cargo.[10] However, general average applies to other losses as well,[11] including sacrifices of part of the vessel such as the cutting away of the mast,[12] and to extraordinary expenses incurred for the joint benefit of the vessel and cargo, such as those sustained in freeing the ship from the strand of a river.[13]

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[6]The principle was incorporated into Islamic legal codes from the eighth century (Khalilieh 2006, p. 160).

[7]See also *The Copenhagen*, 1 Chr. Rob. 289 (1799), in which Lord Stowell of the Court of Admiralty wrote: "General average is for a loss incurred, towards which the whole concern is bound to contribute pro rata, because it was undergone for the general benefit and preservation of the whole" (Lowndes et al. 1912, p. 18).

[8]See also *Star of Hope*, 76 U.S. 203, 228 (1869); *Fowler v. Rathbones*, 79 U.S. 102, 114 (1870); *Hobson v. Lord*, 92 U.S. 397, 404 (1875); *Ralli v. Troop*, 157 U.S. 386, 395 (1895). The earliest general average cases in the U.S. Supreme Court were *Columbian Insurance Co. v. Ashby*, 38 U.S. 331, 338 (1839), and *Barnard v. Adams*, 51 U.S. 270 (1850).

[9]*Columbian Insurance Co. v. Ashby*, 38 U.S. 331, 338 (1839). See also, e.g., *Barnard v. Adams*, 51 U.S. 270 (1850); *Ralli v. Troop*, 157 U.S. 386 (1895).

[10]See *Ralli v. Troop*, 157 U.S. 386, 393 (1895); Rose (1997, p. 5)

[11]For a non-exhaustive list, see Rose (1997, p. 5).

[12]See *Ralli v. Troop*, 157 U.S. 386, 393 (1895).

[13]See *Navigazione Generale Italiana v. Spencer Kellogg & Sons*, 92 F.2d 41 (2d Cir. 1937).

While the law of general average is part of the general maritime law of nations, international maritime interests have created a set of rules to harmonize general average practice around the world (Robertson et al. 2001, p. 426). The first version of these rules was known as the Glasgow Resolutions 1860 (Lowndes et al. 1912, pp. 41-44). The current version is known as the York-Antwerp Rules 2016. These rules have never been adopted by treaty and do not have the force of law; however, they are widely incorporated in bills of lading and courts generally enforce them as binding terms of contract between the parties (Gilmore and Black 1975, pp. 252-253; Robertson et al. 2001, p. 426; Shoenbaum 2011, pp. 256-257).[14]

[TBA: Paragraphs on (i) modern cases of general average and (ii) marine insurance and general average.]

3 The General Average Game

There is a set $\mathcal{N} = \{1, \ldots, N\}$ of $N > 2$ cargo owners. Each owner $i \in \mathcal{N}$ ships one cargo box with unit weight on the same vessel. Each owner i has a utility function U_i that is strictly increasing, strictly concave, and twice differentiable. To capture heterogeneity in risk preferences, we assume $U_i \neq U_j$ for all $i, j \in \mathcal{N}$, $i \neq j$.[15] Without loss of generality, we normalize each owner's initial wealth to zero, apart from her cargo box.

The true value of owner i's cargo box is $t_i \in (0, \bar{t}]$ where $\bar{t} > 0$. The true value t_i is owner i's subjective and private information. The true values are independent and identically distributed according to a probability density function f_t with support $(0, \bar{t}]$. Consequently, with probability one, there are no ties among the true values. This captures heterogeneity in endowments. Let $t = (t_1, \ldots, t_N)$ denote the vector of true values and $T = \sum_{i=1}^{N} t_i$ denote

[14] The York-Antwerp Rules 2016 are available at comitemaritime.org. They have three parts. Two prefatory rules form the first part. The second part comprises seven rules lettered A through G that specify basic principles. The third part contains 23 rules numbered I through XXIII that cover specific types of losses. Because our general average game features the jettison of cargo, we highlight two rules pertaining to cargo lost by sacrifice. First, Rules XVI(a)(i) & XVII(a)(i) together provide that the amount to be allowed as general average, and the contribution to a general average, shall be based on the value at the time of shipment, unless there is a commercial invoice rendered to the receiver, in which case it shall be based on the value at the time of discharge. Second, Rule XIX(b) provides that "[w]here goods have been wrongfully declared at the time of shipment at a value which is lower than their real value, any general average loss or damage shall be allowed on the basis of their declared value, but such goods shall contribute on the basis of their actual value." In our general average game, we consider the situation where the true values are the cargo owners' subjective and private information, and hence we posit that recovery amounts for jettisoned cargo, and contribution amounts for all cargo, are based on the declared values at the time of shipment. This is consistent with Rules XVI(a)(i) & XVII(a)(i), but seemingly inconsistent with Rule XIX(b); however, when cargo values are subjective and private information, Rule XIX(b) is facially inoperable.

[15] The case of heterogeneity is arguably the interesting case, as there is ample evidence of heterogeneity in risk preferences in insurance settings (Cohen and Einav 2007; Einav et al. 2012; Barseghyan et al. 2011, 2013, 2016, 2021), including mutual insurance settings (Mazzocco and Saini 2012; Chiappori et al. 2014).

the total true value of the cargo boxes. For the vector of true values, we sometimes use the notation $t = (t_i, t_{-i})$ where $t_{-i} \in (0, \bar{t}]^{N-1}$ refers to the subvector of true values other than t_i.

At the outset of the venture, the owners privately declare the values of their cargo boxes to the master of the vessel. We assume that the master subsequently publishes the declared values. This assumption is without loss of generality, however, because no further strategic decisions are made in the game. Other than the true values, which are the owners' private information, we assume that all other aspects of the game are common knowledge.

Let $v_i \in (0, \bar{t}]$ denote the declared value of owner i's cargo box, $v = (v_1, \ldots, v_N)$ denote the vector of declared values, and $V = \sum_{i=1}^{N} v_i$ denote the total declared value of the cargo boxes. For the vector of declared values, we sometimes use the notation $v = (v_i, v_{-i})$ where $v_{-i} \in (0, \bar{t}]^{N-1}$ refers to the subvector of declared values other than v_i.

The owners' declarations—and the realization of a random tie-breaking rule r applied to break any ties among them—induce a strict ordering of the cargo boxes. Assume r is distributed according to a probability mass function f_r with support $\Psi(\mathcal{N})$, where $\Psi(\mathcal{N})$ denotes the set of all permutations of N. A realization of r is a randomly selected ordering of the cargo owners.[16] Thus, for any and all sets of tied declarations, a realization of r can be applied to strictly order such declarations. Let $n_i(v, r)$ denote the place of owner i's cargo box in the ascending order of declared values. (For the avoidance of doubt, all references to "the ascending order of declared values" or "in ascending order of declared value" refer to such order with any ties broken.) We assume that the master labels each cargo box with its owner's identity, declared value, and place in the ascending order of declared values.

In an emergency, the master jettisons cargo boxes in ascending order of declared value. Let $\Theta = \{0, \ldots, N\}$ and $\theta \in \Theta$ denote the number of cargo boxes that must be jettisoned in order to save the vessel and the remaining cargo. Assume θ is distributed according to a probability mass function f_θ with support Θ. Owner i's cargo box is jettisoned if and only if $n_i(v, r) \leq \theta$. Let $J_v(v, \theta)$ denote the total declared value of the cargo boxes that are jettisoned (i.e., the sum of their declared values) and $P_v(v, \theta) = J_v(v, \theta)/V$ denote the proportion of the total declared value that is jettisoned. Similarly, let $J_t(t, v, r, \theta)$ denote the total true value of the cargo boxes that are jettisoned (i.e., the sum of their true values) and $P_t(t, v, r, \theta) = J_t(v, \theta)/T$ denote the proportion of the total true value that is jettisoned.[17] Note that while $P_v(v, \theta)$ is common knowledge, $P_t(t, v, r, \theta)$ is unknown to all.

[16] The only requirement that f_r must satisfy is that all permutations must have positive probability.

[17] Note that the total declared value of the cargo boxes that are jettisoned does not depend on r because it only affects the ordering of cargo boxes with equal declared values. By contrast, the total true value of the cargo boxes that are jettisoned does depend on r because, with probability one, cargo boxes with equal declared values have unequal true values.

Under the law of general average, which prescribes proportional sharing of the collective loss, owner i's final wealth equals $v_i - v_i P_v(v,\theta)$ if her cargo box is jettisoned and $t_i - v_i P_v(v,\theta)$ if her cargo box is not jettisoned. That is, owner i's payoff is

$$c_i = \begin{cases} v_i - v_i P_v(v,\theta) & \text{if} \quad n_i(v,r) \leq \theta \\ t_i - v_i P_v(v,\theta) & \text{if} \quad n_i(v,r) > \theta \end{cases}. \tag{1}$$

In what follows, we sometimes refer to the first component of owner i's payoff, v_i or t_i (as the case may be), as her *recovery*, and to the second component, $v_i P_v(v,\theta)$, as her *contribution*. Observe that summing the payoffs across all owners for any given θ, we obtain $(1 - P_t(t,v,r,\theta))T$.[18] Thus, the outcome of the general average game is an allocation among the owners of the total true value of the cargo boxes that are not jettisoned.

4 Equilibrium of the Bayesian Game

Assume cargo owners have expected utility preferences. In this case the general average game is a Bayesian game. Each owner i knows the true value of her cargo box t_i (and all other aspects of the game other than t_{-i}). Her declaration, therefore, is a function $b_i : (0,\bar{t}] \to (0,\bar{t}]$. The declaration function b_i is effectively owner i's strategy. Let $b = (b_1, \ldots, b_N)$ denote the vector of declaration functions. We sometimes use the notation $b = (b_i, b_{-i})$ where b_{-i} refers to the subvector of all declaration functions other than b_i, and we sometimes write b_i as $b_i(\cdot)$ to emphasize that it is a function.[19] In equilibrium, for every owner $i \in \mathcal{N}$, (i) it is *as if* she the declaration functions $b_{-i}(\cdot)$, and hence knows $v_{-i} = b_{-i}(t_{-i})$ for all $t_{-i} \in (0,\bar{t}]^{N-1}$, and (ii) her declaration $v_i = b_i(t_i)$ must maximize her expected payoff given $b_{-i}(\cdot)$.

As we note in section 1, *truthful reporting*, i.e., $b_i(t_i) = t_i$ for all $i \in \mathcal{N}$ and $t_i \in (0,\bar{t}]$, is sufficient, but not necessary, to enable the master to minimize the owners' collective loss. What's necessary is *truthful ordering*, i.e., the owners' declared values must be in the same rank order as their true values.

Definition 1 *The cargo owners' declared values have a* truthful order *if and only if*

$$t_i < t_j \iff b_i(t_i) < b_j(t_j)$$

for all $i, j \in \mathcal{N}$ and $t_i, t_j \in (0,\bar{t}]$.

[18] See section A.1 in the Appendix.

[19] For the same reason, we sometimes write other functions in this way as well—e.g., $U_i(\cdot)$.

The following lemma, the proof of which is set forth in section A.2 of the Appendix, establishes that if the declared values have a truthful order, then every owner must have the same declaration function.

Lemma 1 *If the declared values have a truthful order, then* $b_i(\cdot) = b_j(\cdot)$ *for all* $i, j \in \mathcal{N}$.

We now can state the following result.

Proposition 1 *If cargo owners have expected utility preferences, then there does not exist a Bayesian Nash equilibrium of the general average game in which the owners' declared values have a truthful order.*

Observe that truthful reporting implies truthful ordering, but not vice versa. Hence, if we do not have truthful ordering in equilibrium, then we do not have truthful reporting either.

Corollary 1 *If cargo owners have expected utility preferences, then there does not exist a Bayesian Nash equilibrium of the general average game in which all owners truthfully declare the values of their cargo boxes.*

The proof of proposition 1 is set forth in section A.3 of the Appendix. The following is a sketch of the argument. Assume the owners' declared values have a truthful order. Take any owner $i \in \mathcal{N}$ and any declaration functions $b_{-i}(\cdot)$. Owner i's declaration problem is

$$\max_{v_i \in (0,\bar{t}]} \Pi_i(t_i, v_i, b_{-i}(t_{-i}), r, \theta) = \mathop{E}_{t_{-i},r,\theta} [U_i(c_i(t_i, v_i, b_{-i}(t_{-i}), r, \theta))] \tag{2}$$

where we now write owner i's payoff c_i as $c_i(t_i, v_i, b_{-i}(t_{-i}), r, \theta)$ to make explicit the variables on which it depends and to reflect that the other owners' declared values v_{-i} depend on their true values t_{-i} via their declaration functions $b_{-i}(\cdot)$:

$$c_i(t_i, v_i, b_{-i}(t_{-i}), r, \theta) = \begin{cases} v_i - v_i P_v(v_i, b_{-i}(t_{-i}), \theta) & \text{if} \quad n_i(v_i, v_{-i}, r) \leq \theta \\ t_i - v_i P_v(v_i, b_{-i}(t_{-i}), \theta) & \text{if} \quad n_i(v_i, v_{-i}, r) > \theta \end{cases}.$$

Observe that owner i can compute her expected payoff $\Pi_i(t_i, v_i, b_{-i}(t_{-i}), r, \theta)$ for any declaration v_i as she knows the distributions of t_{-i}, r, and θ. The solution to problem (2) is $v_i^* = b_i^*(t_i)$. However, because $(U_i, t_i) \neq (U_j, t_j)$ for any other owner $j \in \mathcal{N}$, the solution for another owner j is $v_j^* = b_j^*(t_j)$ where $b_j^*(\cdot) \neq b_i^*(\cdot)$, which contradicts lemma 1. Hence, the owners' declared values cannot have a truthful order in equilibrium, and so we do not have truthful reporting in equilibrium.

5 Equilibrium of the Maxmin Game

In the previous section, we showed that there does not exist a truthful equilibrium of the general average game if cargo owners have expected utility preferences. In this section, by contrast, we establish the following result.

Proposition 2 *If cargo owners have maxmin utility preferences, then truthful declarations by all owners is the unique Nash equilibrium of the general average game.*

Assume owners have maxmin utility preferences. Given any declarations v_{-i} by the other owners, owner i's declaration problem is

$$\max_{v_i \in (0,\bar{t}]} \min_{\substack{v_{-i} \in (0,\bar{t}]^{N-1} \\ r \in \Psi(\mathcal{N}) \\ \theta \in \Theta}} \begin{cases} U_i(v_i - v_i P_v(v_i, v_{-i}, \theta)) & \text{if} \quad n_i(v_i, v_{-i}, r) \leq \theta \\ U_i(t_i - v_i P_v(v_i, v_{-i}, \theta)) & \text{if} \quad n_i(v_i, v_{-i}, r) > \theta \end{cases}.$$

In other words, the owner chooses her declaration, v_i, to maximize her payoff assuming the worst-case combination of the declarations by the other owners v_{-i}, the realization of the tie-breaking rule r, and the number of cargo boxes that are jettisoned θ.

Observe that if $\theta \in \{0, N\}$, the owner's utility does not depend on her declaration v_i. After all, if no cargo boxes are jettisoned then her payoff is t_i, and if all cargo boxes are jettisoned then her payoff is zero.[20] Thus, defining $\overline{\Theta} = \{1, \ldots, N-1\}$, the nondegenerate problem is

$$\max_{v_i \in (0,\bar{t}]} \min_{\substack{v_{-i} \in (0,\bar{t}]^{N-1} \\ r \in \Psi(\mathcal{N}) \\ \theta \in \overline{\Theta}}} \begin{cases} U_i(v_i - v_i P_v(v_i, v_{-i}, \theta)) & \text{if} \quad n_i(v_i, v_{-i}, r) \leq \theta \\ U_i(t_i - v_i P_v(v_i, v_{-i}, \theta)) & \text{if} \quad n_i(v_i, v_{-i}, r) > \theta \end{cases}. \tag{3}$$

Looking at problem (3), we can see that, given v_i, whether or not owner i's cargo box is jettisoned, the worst-case combination of v_{-i} and θ is the combination that yields the maximum value of $P(v_i, v_{-i}, \theta)$. The following lemma establishes that this value is $(N-1)/N$.

Lemma 2 *Take any $v_i \in (0, \bar{t}]$. Suppose $v_{-i} \in (0, \bar{t}]^{N-1}$, $r \in \Psi(\mathcal{N})$, and $\theta \in \overline{\Theta}$. Then the maximum value that $P_v(v_i, v_{-i}, \theta)$ can achieve is $(N-1)/N$. This value is achieved by setting $v_j = v_i$ for all $j \in \mathcal{N}$, $j \neq i$, and $\theta = N-1$.*

Proof See section A.4 in the Appendix.

[20]Formally, if $\theta = 0$ then $P_v(v_i, v_{-i}, \theta) = 0$ and hence $t_i - v_i P_v(v_i, v_{-i}, \theta) = t_i$, and if $\theta = N$ then $P_v(v_i, v_{-i}, \theta) = 1$ and hence $v_i - v_i P_v(v_i, v_{-i}, \theta) = 0$.

In what follows, we consider two collectively exhaustive cases, $v_i \geq t_i$ and $v_i \leq t_i$, and apply the lemma 2 to show that in each case the solution to problem (3) is $v_i^* = t_i$.

First, suppose the owner declares $v_i \geq t_i$. For any given combination of v_i and θ, this implies $v_i - v_i P_v(v_i, v_{-i}, \theta) \geq t_i - v_i P_v(v_i, v_{-i}, \theta)$. It follows that the worst-case combination of v_i, r, and θ is the combination that yields $P_v(v_i, v_{-i}, \theta) = (N-1)/N$ and $n_i(v_i, v_{-i}, r) > \theta$. In this case, problem (3) amounts to

$$\max_{v_i \in [t_i, \bar{t}]} U_i \left(t_i - v_i \frac{N-1}{N} \right).$$

Observe that $t_i - v_i[(N-1)/N]$ is strictly decreasing in v_i. Because $U_i(\cdot)$ is strictly increasing, this implies that the solution to the owner's problem in this case is $v_i^* = t_i$.

Next, suppose the owner declares $v_i \leq t_i$. For any given combination of v_i and θ, this implies $v_i - v_i P_v(v_i, v_{-i}, \theta) \leq t_i - v_i P_v(v_i, v_{-i}, \theta)$. It follows that the worst-case combination of v_i, r, and θ is the combination that yields $P_v(v_i, v_{-i}, \theta) = (N-1)/N$ and $n_i(v_i, v_{-i}, r) \leq \theta$. In this case, problem (3) amounts to

$$\max_{v_i \in (0, t_i]} U_i \left(v_i - v_i \frac{N-1}{N} \right).$$

Observe that $v_i - v_i[(N-1)/N]$ is strictly increasing in v_i. Because $U_i(\cdot)$ is strictly increasing, this implies that the solution to the owner's problem in this case is also $v_i^* = t_i$.

The foregoing establishes that making a truthful declaration is the unique solution to problem (3), assuming owner i has maxmin utility preferences. Because owner i is arbitrary, this implies that if cargo owners have maxmin utility preferences, then truthful declarations by all owners is the unique Nash equilibrium of the general average game.

Remark The key to understanding owner i's maxmin behavior lies in her "conjecture" that whatever declaration v_i she chooses, the other owners and nature will "conspire" to minimize her payoff. This entails them choosing v_{-i} and θ to maximize $P_v(v_i, v_{-i}, \theta)$, which increases her contribution, $v_i P_v(v_i, v_{-i}, \theta)$, all else equal. By lemma 2, these values are $v_{-i} = v_i$ and $\theta = (N-1)/N$. Moreover, owner i conjectures that (i) if she overdeclares then the tie-breaking rule will result in her cargo box not being jettisoned, in which case she receives her true value minus her contribution (i.e., she is denied the benefit of her overdeclaration (overrecovery) but suffers the cost (higher contribution)), and (ii) if she underdeclares then the tie-breaking rule will result in her cargo box being jettisoned, in which case she receives her declared value minus her contribution (i.e., she suffers the cost of her underdeclaration (underrecovery) which exceeds the benefit (lower contribution)). In the former case she gains by decreasing her declaration to her true value (which does not affect her recovery but decreases her contribution), and in the latter case she gains by increasing her declaration to her true value (which increases her recovery by more than it increases her contribution).

6 Pareto Efficiency

Recall that the outcome of the general average game is an allocation among the cargo owners of the total true value of the cargo boxes that are not jettisoned, $(1 - P_t(t, v, r, \theta))T$. In this section, we investigate the conditions under which an allocation prescribed by the law of general average, to which we refer as a general average allocation, is Pareto efficient.

As a threshold matter, Pareto efficiency requires that the proportion of the total true value that is jettisoned, $P_t(t, v, r, \theta)$, is the minimum necessary to save the vessel and the remaining cargo. We refer to this requirement as *resource efficiency*. Given that cargo boxes are jettisoned in ascending order of declared value, resource efficiency is attained when the declared values have a truthful order.

6.1 Expected Utility Preferences

Suppose cargo owners have expected utility preferences. Recall that in this case there does not exist a Bayesian Nash equilibrium of the general average game in which the declared values have a truthful order. An immediate implication is that the law of general average does not yield a Pareto efficient allocation in equilibrium, because without truthful ordering (let alone truthful reporting) resource efficiency is not attained. Even with truthful reporting, however, a general average allocation, which ipso facto entails proportional loss sharing, is Pareto efficient if and only if owners have identical (up to a positive scalar) CRRA utility functions. We prove this claim in section A.5 of the Appendix.

The following proposition recaps the foregoing.

Proposition 3 *If cargo owners have expected utility preferences, then the law of general average does not yield a Pareto efficient allocation in equilibrium, because there is no Bayesian Nash equilibrium of the general average game in which there is a truthful ordering of declared values (let alone truthful reporting), and hence resource efficiency is not attained in equilibrium. Even assuming truthful reporting, a general average allocation is Pareto efficient if and only if owners have identical (up to a positive scalar) CRRA utility functions.*

6.2 Maxmin Utility Preferences

Now suppose cargo owners have maxmin utility preferences. Recall that in this case truthful reporting is the unique Nash equilibrium of the general average game. In equilibrium, therefore, the law of general average yields the following allocation as a function of θ:

$$c_i(\theta) = (1 - P_t(t, v, r, \theta))t_i \qquad \forall\, i \in \mathcal{N},\ \forall\, \theta \in \Theta.$$

Call this allocation c^* and denote its components by $c_i^*(\theta)$.

With maxmin preferences, the utility that owner i derives from any allocation c is the utility assuming the worse-case state, $\min_{\theta\in\Theta} U_i(c_i(\theta))$. Let $\overline{\theta}$ denote the worse-case state.[21] Then the utility that owner i derives from an allocation c is $U_i(c_i(\overline{\theta}))$.

Given resource efficiency, which is implied by truthful reporting, an allocation is Pareto efficient if and only if there does not exist a reallocation of resources that makes at least one owner better off without making at least one other owner worse off. Take allocation c^*. The only utility-relevant components of c^* are the payoffs in state $\overline{\theta}$,

$$c_i^*(\overline{\theta}) = (1 - P_t(t, v, r, \theta))t_i \qquad \forall\, i \in \mathcal{N},$$

and the only relevant resource constraint is the one pertaining to state $\overline{\theta}$,

$$\sum_{i=1}^{N} c_i^*(\overline{\theta}) = (1 - P_t(t, v, r, \theta))T.$$

Because c^* exhausts all available resources in each state,[22] it follows that any reallocation c' which increases the payoff in state $\overline{\theta}$ to owner i, $c_i'(\overline{\theta}) > c_i^*(\overline{\theta})$, necessarily decreases the payoff in state $\overline{\theta}$ to some other owner j, $c_j'(\overline{\theta}) < c_j^*(\overline{\theta})$, in order to obey the resource constraint for state $\overline{\theta}$. Because $U_i(\cdot)$ is strictly increasing, this implies that there does not exist a reallocation of resources that makes at least one owner better off without making at least one other owner worse off. Hence, allocation c^* is Pareto efficient. To recap:

Proposition 4 *If cargo owners have maxmin utility preferences, then the law of general average yields a Pareto efficient allocation in equilibrium.*

[21] Lemma 2 establishes that $\overline{\theta} = N - 1$. The argument that follows, however, applies given any value of $\overline{\theta}$.

[22] For all $\theta \in \Theta$, $\sum_{i=1}^{N} c_i^*(\theta) = \sum_{i=1}^{N}(1 - P_t(t, v, r, \theta))t_i = (1 - P_t(t, v, r, \theta))T$.

7 Discussion

[This section will contain a concluding discussion in which we (i) compare and contrast the results of the general average game with those of the *La Crema* game in (Cabrales et al. 2003), (ii) suggest why the maxmin criterion may be a reasonable decision rule in the context of the law of general average, and (iii) point to directions for future research (e.g., examining alternative sharing rules). With respect to (ii), we will argue that while weighing states of nature and maximization of expected utility may be reasonable when a decision maker has a credible basis for placing a subjective probability distribution on unknown features of the decision problem, when this is not the case—i.e., when the decision maker faces ambiguity, or Knightian uncertainty, which may very well be the case in the general average context, where each voyage and its risks may be idiosyncratic/singular—a decision maker may reasonably evaluate actions by the worst utility that they may yield and choose an action that yields that least-bad worst utility (Manski 2013). We will also highlight that the maxmin criterion is a deeply rooted idea in social science. Wald (1950) developed it as a solution of a statistical decision problem when a prior probability distribution is unknown. Rawls (1971) invoked it as part of a normative theory of justice. Gilboa and Schmeidler (1989) proposed it as a model of choice under uncertainty when the decision maker has too little information to form a prior and is uncertainty averse. In the end, the maxmin criterion may make quite a bit of sense for a cargo owner that must make a decision regarding what to declare under a veil of ignorance about nature's true distribution.]

Appendix

A.1 Summation of Payoffs

Take any $\theta \in \Theta$. Let $\mathcal{J}(\theta) \subseteq \mathcal{N}$ denote the set of owners whose cargo boxes are jettisoned. Summing the payoffs in equation (1) across all owners, we have

$$\begin{aligned}
&\sum_{i \in \mathcal{J}(\theta)} v_i - v_i P_v(v,\theta) + \sum_{i \in \mathcal{N} \setminus \mathcal{J}(\theta)} t_i - v_i P_v(v,\theta) \\
&= \sum_{i \in \mathcal{J}(\theta)} v_i + \sum_{i \in \mathcal{N} \setminus \mathcal{J}(\theta)} t_i - \sum_{i \in \mathcal{N}} v_i P_v(v,\theta) \\
&= J_v(v,\theta) + (1 - P_t(t,v,r,\theta))T - J_v(v,\theta) = (1 - P_t(t,v,r,\theta))T.
\end{aligned}$$

A.2 Proof of Lemma 1

[TBA]

A.3 Proof of Proposition 1

[TBA]

A.4 Proof of Lemma 2

Take any $v_i \in (0, \bar{t}]$. Suppose $v_{-i} \in (0, \bar{t}]^{N-1}$, $r \in \Psi(\mathcal{N})$, and $\theta \in \overline{\Theta}$ are such that owner i's cargo box is jettisoned. Let $\mathcal{J}(\theta) \subseteq \mathcal{N}$ denote the set of owners whose cargo boxes are jettisoned. Note that $i \in \mathcal{J}(\theta)$. Then

$$P_v(v_i, v_{-i}, \theta) = \frac{v_i + \sum\limits_{j \in \mathcal{J}(\theta), j \neq i} v_j}{v_i + \sum\limits_{j \in \mathcal{N}, j \neq i} v_j}. \tag{A.1}$$

Observe that the numerator of equation (A.1) is strictly increasing in θ while the denominator is independent of θ. Hence, to maximize equation (A.1), we must set $\theta = N - 1$. Given this, equation (A.1) becomes

$$P_v(v_i, v_{-i}, \theta) = \frac{v_i + \sum\limits_{j \in \mathcal{J}(\theta), j \neq i, j \neq N} v_j}{v_i + v_N + \sum\limits_{j \in \mathcal{N}, j \neq i, j \neq N} v_j}. \tag{A.2}$$

Note that equation (A.2) is strictly increasing in the summation term in the numerator. Thus, to maximize (A.2), we must set each declared value in the summation equal to $\bar{t}$, which implies that we also must set v_N equal to $\bar{t}$ (because v_N is the highest declared value), and we conclude that the maximum value of $P_v(v_i, v_{-i}, \theta)$ in this case is

$$P_v(v_i, v_{-i}, \theta) = \frac{v_i + (N-2)\bar{t}}{v_i + (N-1)\bar{t}}. \tag{A.3}$$

Take the same $v_i \in (0, \bar{t}]$. But now suppose $v_{-i} \in (0, \bar{t}]^{N-1}$, $r \in \Psi(\mathcal{N})$, and $\theta \in \overline{\Theta}$ are such that owner i's cargo box is not jettisoned. Let $\mathcal{J}(\theta) \subseteq \mathcal{N}$ denote the set of owners whose cargo boxes are jettisoned. Note that now $i \notin \mathcal{J}(\theta)$. Then

$$P_v(v_i, v_{-i}, \theta) = \frac{v_i + \sum\limits_{j \in \mathcal{J}(\theta)} v_j}{v_i + \sum\limits_{j \in \mathcal{N}, j \neq i} v_j}. \tag{A.4}$$

Observe that the numerator of equation (A.4) is strictly increasing in θ while the denominator is independent of θ. Hence, to maximize equation (A.4), we must set $\theta = N - 1$. Given this, the summations in the numerator and denominator are both the summation of all declared values other than v_i. Thus, to maximize equation (A.4), we must set these declared values as high as possible without violating the condition $n_i(v_i, v_{-i}, r) > \theta$. We can do this by setting them all equal to v_i. (With all declared values equal, the tie-breaking rule can be set such that owner i's cargo box is the only one not jettisoned.) We therefore conclude that the maximum value of $P_v(v_i, v_{-i}, \theta)$ in this case is

$$P_v(v_i, v_{-i}, \theta) = \frac{\sum\limits_{j \in \mathcal{N}, j \neq i} v_j}{v_i + \sum\limits_{j \in \mathcal{N}, j \neq i} v_j} = \frac{(N-1)v_i}{v_i + (N-1)v_i} = \frac{(N-1)}{N}. \tag{A.5}$$

Note that because the maximum value of $P_v(v_i, v_{-i}, \theta)$ set forth in equation (A.5), namely $(N-1)/N$, is achieved with all declared values being equal, it follows that it can be achieved with the tie-breaking rule yielding either that owner i's cargo box is not the one jettisoned or that owner i's cargo box is the one jettisoned. Therefore, to conclude the proof, it is sufficient to show that $(N-1)/N$ is weakly greater than the maximum value of $P_v(v_i, v_{-i}, \theta)$ set forth in equation (A.3). Indeed, this is immediate by noting that equation (A.3) is strictly increasing in v_i and, in fact, is equal to $(N-1)/N$ when $v_i = \bar{t}$.

A.5 Proof of Claim in Section 6

In this section we prove the claim, made in section 6, that if cargo owners have expected utility preferences, then even with truthful reporting, a general average allocation is Pareto efficient if and only if owners have identical (up to a positive scalar) CRRA utility functions.

Assume cargo owners have expected utility preferences. Let $c_i(\theta)$ denote the payoff to owner i in state θ. An allocation is an array $c = [c_i(\theta)]_{i \in \mathcal{N}, \theta \in \Theta}$ of payoffs to all owners $i \in \mathcal{N}$ in all states $\theta \in \Theta$. With truthful reporting, the law of general average prescribes the following allocation:

$$c_i(\theta) = (1 - P_t(t, v, r, \theta))t_i \qquad \forall\, i \in \mathcal{N},\ \forall\, \theta \in \Theta.$$

Thus, a truthful general average allocation is characterized by

$$\frac{c_j(\theta)}{c_i(\theta)} = \frac{t_j}{t_i} \qquad \forall\, i, j \in \mathcal{N},\ \forall\, \theta \in \Theta, \tag{A.6}$$

$$\frac{c_i(\theta'')}{c_i(\theta')} = \frac{1 - P_t(t, v, r, \theta'')}{1 - P_t(t, v, r, \theta')} \qquad \forall\, i \in \mathcal{N},\ \forall\, \theta', \theta'' \in \Theta. \tag{A.7}$$

Give the assumptions on U_i, the set of Pareto efficient allocations comprises the solutions to the planner's problem with positive Pareto weights:

$$\max_c \sum_{i=1}^{N} \sum_{\theta=0}^{N} \alpha_i \left[U_i(c_i(\theta))\mu(\theta) \right], \qquad \alpha_1, \ldots, \alpha_N > 0,$$

subject to the resource constraints

$$\sum_{i=1}^{N} c_i(\theta) = (1 - P_t(t, v, r, \theta))T \qquad \forall\, \theta \in \Theta,$$

which are satisfied here. The necessary and sufficient first-order conditions are

$$U_i'(c_i(\theta)) = \frac{\lambda_\theta}{\mu(\theta)\alpha_i} \qquad \forall\, i \in \mathcal{N},\ \forall\, \theta \in \Theta,$$

where λ_θ denotes the Lagrange multiplier pertaining to the θ-constraint. It follows that the set of Pareto efficient allocations is characterized by

$$\frac{U_i'(c_i(\theta))}{U_j'(c_j(\theta))} = \frac{\alpha_j}{\alpha_i} \qquad \forall\, i, j \in \mathcal{N},\ \forall\, \theta \in \Theta, \tag{A.8}$$

$$\frac{U_i'(c_i(\theta'))}{U_i'(c_i(\theta''))} = \frac{\lambda_{\theta'}\mu(\theta'')}{\lambda_{\theta''}\mu(\theta')} \qquad \forall\, i \in \mathcal{N},\ \forall\, \theta', \theta'' \in \Theta. \tag{A.9}$$

Suppose owners have identical (up to a positive scalar) CRRA utility functions. That is,

$$U_i(x) = \begin{cases} \beta_i \frac{x^{1-\eta}}{1-\eta} & \text{if} \quad \eta \neq 1 \\ \beta_i \ln(x) & \text{if} \quad \eta = 1 \end{cases} \qquad \forall\, i \in \mathcal{N},$$

where $\beta_i > 0$ and $\eta \geq 0$. Then conditions (A.8) and (A.9) become

$$\frac{c_j(\theta)}{c_i(\theta)} = \left[\frac{\alpha_j}{\alpha_i}\right]^{\frac{1}{\eta}} \qquad \forall\, i, j \in \mathcal{N},\ \forall\, \theta \in \Theta, \tag{A.10}$$

$$\frac{c_i(\theta'')}{c_i(\theta')} = \left[\frac{\lambda_{\theta'}\mu(\theta'')}{\lambda_{\theta''}\mu(\theta')}\right]^{\frac{1}{\eta}} \qquad \forall\, i \in \mathcal{N},\ \forall\, \theta', \theta'' \in \Theta. \tag{A.11}$$

Comparing conditions (A.6)-(A.7) and conditions (A.10)-(A.11), we can see that there exist positive Pareto weights and Lagrange multipliers such that the two pairs of conditions are equivalent. Moreover, this is not the case for utility functions outside the CRRA family, because only CRRA utility implies that payoff ratios across owners depend only on relative wealth levels and that payoff ratios across states depend only on relative shadow prices.

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